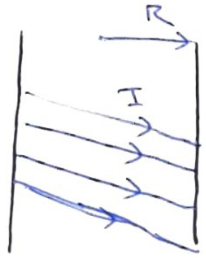
ELECTRICITY AND MAGNETISM 21.05.2021

Test 5

1]



(1) $\vec{B} = \mu_0 n I \hat{z}$

There is no $\vec{E}$ field inside the solenoid

$$u = \frac{1}{2}\left(\underbrace{\varepsilon_0 E^2}_{0} + \frac{1}{\mu_0} B^2\right)$$

so $$u = \frac{1}{2}\left(\frac{1}{\mu_0} B^2\right) = \frac{1}{2}\left(\frac{1}{\mu_0}\left(\mu_0 n I\right)^2\right)$$

$$u = \frac{1}{2}\left(\mu_0 n^2 I^2\right) \Rightarrow u = \frac{\mu_0 n^2 I^2}{2}$$

(2) Current is changing. so there is now an electric field inside the solenoid

$$\vec{B}(t) = \mu_0 n I(t) \hat{z}$$

$$\cancel{\phi} \ \phi = B(t) \pi R^2$$

$$E(\cancel{\pi R^2}^{\,2\pi R}) = \frac{d}{dt}\left(\pi R^2 B(t)\right) \Rightarrow E(\cancel{\pi R^2}^{\,(2\pi R)}) = \cancel{\frac{d}{dt}} R \cancel{\pi R^2} \frac{d}{dt}\left(B(t)\right)$$

from $\oint E \cdot dl$

$$\vec{E} = \frac{d}{dt}\left(\frac{sB(t)}{2}\right) \Rightarrow \vec{E} = \frac{d}{dt}\left(\frac{\mu_0 n I(t)}{2}\right) s$$

$$\vec{E} = \frac{\mu_0 n}{2} \frac{dI}{dt} s \hat{\varphi} \quad //$$

(3) $$\vec{S} = \frac{1}{\mu_0}\left(\vec{E} \times \vec{B}\right)$$

$$\vec{S} = \frac{1}{\mu_0}\left[\left(\frac{\mu_0 n}{2} \frac{dI}{dt} s \hat{\varphi}\right) \times \left(\mu_0 n I(t) \hat{z}\right)\right]$$

$$I \frac{dI}{dt} = \frac{1}{2} \frac{d(I^2)}{dt}$$

$$\vec{S} = \frac{1}{\cancel{\mu_0}}\left[\left(\frac{\mu_0^{\cancel{2}} n^2}{2} s I \frac{dI}{dt}\right) \hat{\theta}\right]$$

$$\vec{S} = \left[\frac{\mu_0 n^2}{2} s \cdot \frac{1}{2} \frac{d(I^2)}{dt}\right] \hat{\theta} \Rightarrow \vec{S} = \frac{\mu_0 n^2 s}{4} \frac{d(I^2)}{dt} \hat{\theta} \quad //$$

ELECTRICITY AND MAGNETISM

Test 5

21. 05. 2021

1] (4) For local energy conservation to hold,

$$\frac{du}{dt} = -\nabla \cdot S$$

$$u = \frac{\mu_0 n^2}{2}(I(t))^2$$

$$\frac{\partial u}{\partial t} = \frac{d}{dt}\left(\frac{\mu_0 n^2}{2}(I(t))^2\right)$$

$$= \frac{\mu_0 n^2}{2}\frac{d}{dt}(I(t))^2$$

so $$\frac{\partial u}{\partial t} = \frac{\mu_0 n^2}{2}\frac{\partial}{\partial t}(I^2)$$

$$\cancel{\vec{S} = \frac{\mu_0 n^2 s}{4}\frac{d}{dt}[I(t)]^2}$$

$$\vec{S} = \frac{\mu_0 n^2 s}{4}\frac{d}{dt}(I(t)^2)$$

For ~~convenece~~ convenience, let $I(t) = I$

$$\vec{S} = \frac{\mu_0 n^2 s}{4}\frac{\partial}{\partial t}(I^2)$$

$$\nabla \cdot S = \frac{\cancel{2}\mu_0 n^2}{\cancel{4}}\frac{d}{dt}(I^2)$$

$$\nabla \cdot S = -\frac{\mu_0 n^2}{2}\frac{d}{dt}(I^2)$$

$$-\nabla \cdot S = \frac{\mu_0 n^2}{2}\frac{d}{dt}(I^2)$$

so we see that

$$\frac{\partial u}{\partial t} = \frac{\mu_0 n^2}{2}\frac{\partial}{\partial t}(I^2) \quad \text{and} \quad -\nabla \cdot S = \frac{\mu_0 n^2}{2}\frac{\partial}{\partial t}(I^2)$$

and so

$$\frac{\partial u}{\partial t} = -\nabla \cdot S$$

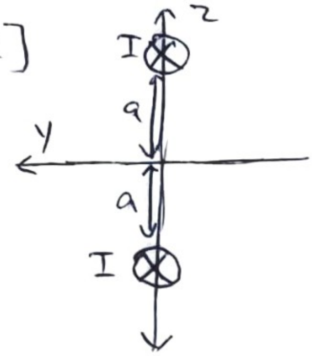
//

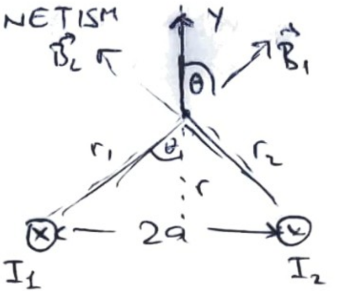
Local energy conservation law holds!

ELECTRICITY AND MAGNETISM 21.05.2021

Test 5

2]





(1)

$$B_1 = \frac{\mu_0 I}{2\pi r_1}\hat{x} \qquad B_2 = \frac{\mu_0 I}{2\pi r_2}\hat{x}$$

we want it in the $\hat{y}$ direction,

so $$\vec{B} = \frac{\mu_0 I}{2\pi r_1}\cos\theta + \frac{\mu_0}{2\pi r_2}\cos\theta$$

$$\cos\theta = \frac{r}{\sqrt{r^2+a^2}}$$

$$\vec{B} = \cancel{22}\,\frac{2\mu_0 I}{2\pi}\cos\theta\,\frac{1}{(r^2+a^2)}\,\hat{y}$$

$$\vec{B} = \frac{\mu_0 I}{\pi}\frac{r}{\sqrt{r^2+a^2}}\frac{1}{r^2+a^2} \Rightarrow \vec{B} = \frac{\mu_0 I}{\pi}\frac{r}{(r^2+a^2)^{3/2}}\hat{y} \quad //$$

(2) No electric field component,

so $$T_{ij} = \frac{1}{\mu_0}\left(B_iB_j - \frac{1}{2}\delta_{ij}B^2\right)$$

No off-diagonal terms, so

$$T_{ii} = \frac{1}{\mu_0}\left(B_iB_i - \frac{1}{2}B^2\right)$$

$$T_{xx} = \frac{1}{\mu_0}\Big(\underbrace{B_xB_x}_{0} - \frac{1}{2}\left(\frac{\mu_0 I}{\pi}\frac{r}{(r^2+a^2)^{3/2}}\right)^2\Big)$$

$$T_{xx} = \frac{-1}{2\mu_0}\left(\frac{\mu_0^2 I^2}{\pi^2}\frac{y^2}{(y^2+a^2)^2}\right)$$ → y because r is in the y direction: $r = y$

$$T_{yy} = \frac{1}{\mu_0}\Big(\underbrace{B_yB_y}_{0} - \frac{1}{2}\left(\frac{\mu_0 I}{\pi}\frac{y}{(y^2+a^2)^{3/2}}\right)^2\Big)$$

$$T_{yy} = \frac{-1}{2\mu_0}\left(\frac{\mu_0^2 I^2}{\pi^2}\frac{y^2}{(y^2+a^2)^2}\right)$$

$$T_{zz} = \frac{1}{\mu_0}\left(B_zB_z - \frac{1}{2}B^2\right) = \frac{1}{\mu_0}\left(\frac{1}{2}B^2\right) = \frac{1}{2\mu_0}\left(\frac{\mu_0^2 I^2}{\pi^2}\frac{y^2}{(y^2+a^2)^2}\right)$$

So we can write

$$\overleftrightarrow{T} = \frac{\mu_0 I^2 y^2}{2\pi^2(y^2+a^2)^2}\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad //$$

(3) ELECTRICITY AND MAGNETISM — 21 Oct 2021

Test 5

2] (3) $\vec{f} = \oint_s (\overleftrightarrow{T}\, d\vec{a})_z$

$$(\overleftrightarrow{T} \cdot da)_z = \underbrace{T_{zx}\, da_x + T_{zy}\, da_y}_{0} + T_{zz}\, da_z \qquad da_z = dx\,dy$$

$$(\overleftrightarrow{T} \cdot da)_z = T_{zz}\, dx\,dy$$

$$= \frac{\mu_0 I^2 y^2}{2\pi^2 (a^2+y^2)^2}\, dx\,dy$$

$$\oint (\overleftrightarrow{T} \cdot da)_z = \int_{-\infty}^{\infty}\int_{L/2}^{L/2} \frac{\mu_0 I^2 y^2}{2\pi^2 (a^2+y^2)^2}\, dx\,dy$$

$$= \frac{\mu_0 I^2}{2\pi^2} \int_{-L/2}^{L/2}\int_{-\infty}^{\infty} \frac{y^2}{(a^2+y^2)^2}\, dy\,dx$$

$$= \frac{\mu_0 I^2}{2\pi^2} \int_{-L/2}^{L/2} \frac{\pi}{2a}\, dx \Rightarrow \frac{\mu_0 I^2}{2\pi^2} \cdot \frac{\pi}{2} \int_{-L/2}^{L/2} \frac{1}{a}\, da$$

$$\vec{f} = \frac{\mu_0 I^2}{4\pi} \cancel{\ln\left(\frac{L/2}{-L/2}\right)} \ln\left(\frac{L}{2}\right) \cancel{- \ln\left(-\frac{L}{2}\right)}$$

$$\vec{f} = \frac{\mu_0 I^2}{4\pi} \ln\left(\frac{L}{2}\right) \qquad L = 2a$$

$$\vec{f} = \frac{\mu_0 I^2}{4\pi} \ln\left(\frac{2a}{2}\right) \Rightarrow \vec{f} = \frac{\mu_0 I^2}{4\pi} \ln(a)$$

ELECTRICITY AND MAGNETISM

Test 5

21.05.2021

3] amplitude = E_0 angular frequency ω $\delta = 0$

origin to point (1, 1, 0)

(1) First, let $\vec{k} = \frac{\omega}{c}\hat{x} + \frac{\omega}{c}\hat{y} + \frac{\omega}{c}\hat{z}$

→ the point has no z component,

so $\vec{k} = \frac{\omega}{c}\hat{x} + \frac{\omega}{c}\hat{y}$ but $|\vec{k}|$ must be $\frac{\omega}{c}$, which is not possible ~~-~~ at the moment

$|\vec{k}| = \sqrt{\left(\frac{\omega}{c}\right)^2 + \left(\frac{\omega}{c}\right)^2} = \frac{\omega}{c}\sqrt{2}$, so I can adapt my original $\vec{k}$ to get rid of the extra $\sqrt{2}$

So then,

$$\vec{k} = \frac{\omega}{\sqrt{2}c}\hat{x} + \frac{\omega}{\sqrt{2}c}\hat{y} \Rightarrow \vec{k} = \frac{\omega}{c}\left(\frac{\hat{x}+\hat{y}}{\sqrt{2}}\right)$$ //

The polarisation is parallel to the xy plane,

so $\vec{n} = \alpha\hat{x} + \beta\hat{y}$

we know that $\vec{k}\cdot\vec{n} = 0$ (they are perpendicular)

so $\left(\frac{\omega}{\sqrt{2}c}\hat{x} + \frac{\omega}{\sqrt{2}c}\hat{y}\right)\left(\alpha\hat{x} + \beta\hat{y}\right) = 0 \Rightarrow \frac{\omega}{\sqrt{2}c}\alpha + \frac{\omega}{\sqrt{2}c}\beta = 0$

so $\alpha = -\beta$

$|\hat{n}| = 1$, so $\sqrt{\alpha^2+\beta^2} = 1 \Rightarrow \sqrt{\alpha^2+\alpha^2} = 1 \Rightarrow \alpha = \frac{1}{\sqrt{2}}$

so $\vec{n} = \frac{1}{\sqrt{2}}\hat{x} - \frac{1}{\sqrt{2}}\hat{y}$

$$\hat{n} = \frac{\vec{n}}{|\hat{n}|} = \hat{n} = \vec{n} = \frac{\hat{x} - \hat{y}}{\sqrt{2}}$$ //

(2) $\vec{E}(\vec{r},t) = E_0\cos(\vec{k}\cdot\vec{r} - \omega t)\hat{n}$ (no δ because $\delta = 0$)

$\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$

$\vec{k}\cdot\vec{r} = \left(\frac{\omega}{c\sqrt{2}}\hat{x} + \frac{\omega}{c\sqrt{2}}\hat{y}\right)\cdot\left(x\hat{x} + y\hat{y} + z\hat{z}\right)$

$= \frac{\omega}{c\sqrt{2}}x + \frac{\omega}{c\sqrt{2}}y \Rightarrow \frac{\omega}{c\sqrt{2}}(x+y)$

So then

~~$\vec{E}(\vec{r},t) =$~~

$$\vec{E}(\vec{r},t) = E_0\cos\left(\frac{\omega}{c\sqrt{2}}(x+y) - \omega t\right)\left(\frac{\hat{x}-\hat{y}}{\sqrt{2}}\right) \Rightarrow \frac{E_0}{\sqrt{2}}\cos\left(\frac{\omega}{c\sqrt{2}}(x+y) - \omega t\right)(\hat{x}-\hat{y})$$ //

(3) ~~td~~ $\vec{B}(\vec{r},t) = B_0 \cos(\vec{k}\cdot\vec{r} - \omega t)(\hat{k} \times \hat{n})$

21.05.2024

we know that $B_0 = \frac{1}{c} E_0$

so $\vec{B}(\vec{r},t) = \frac{1}{c} E_0 \cos\left(\frac{\omega}{c\sqrt{2}}(x+y) - \omega t\right)(\hat{k} \times \hat{n})$

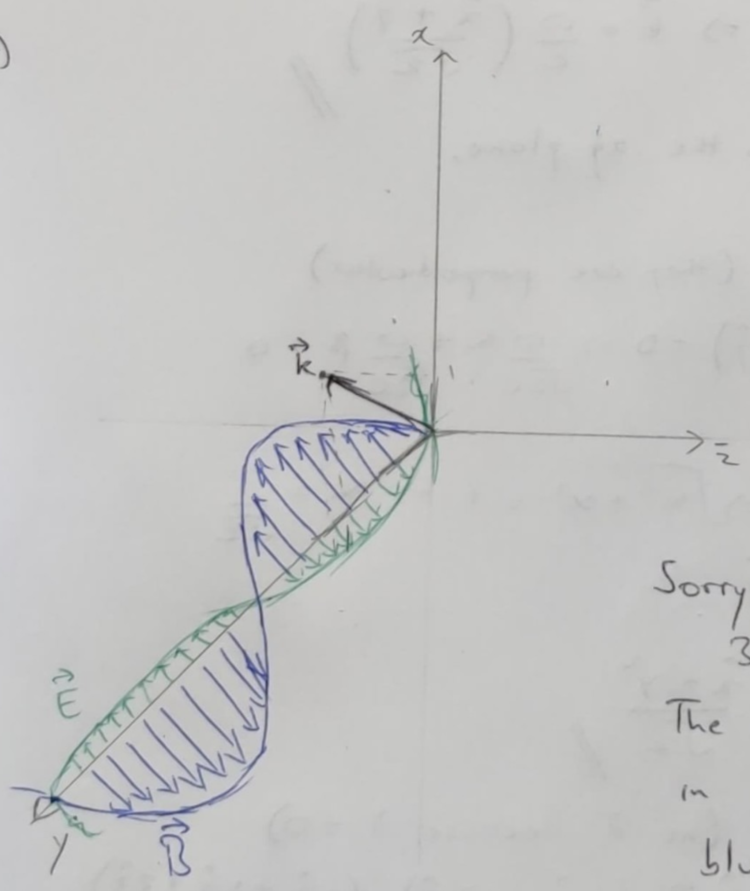
$$\hat{k} \times \hat{n} = \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 1 & 1 & 0 \\ 1 & -1 & 0 \end{vmatrix} = \frac{1}{2}\left[\hat{x}(0) + -\hat{y}(0) + \hat{z}(-1-1)\right]$$

$$= \frac{1}{2}(-2\hat{z})$$

So $\vec{B}(\vec{r},t) = \frac{1}{2c} E_0 \cos\left(\frac{\omega}{c\sqrt{2}}(x+y) - \omega t\right)(-2\hat{z})$

$$\Rightarrow \vec{B}(\vec{r},t) = -\frac{E_0}{c} \cos\left(\frac{\omega}{c\sqrt{2}}(x+y) - \omega t\right)\hat{z}$$

(4)



Sorry I can't sketch well in 3D

The green wave ($\vec{E}$) is moving in the x+y plane, while the blue line ($\vec{B}$) is moving in only the $\hat{y}$ ~~p~~ axis

ELECTRICITY AND MAGNETISM

21.05.2021

TEST 5

4] $M = 419000\,kg$ $A = 2500\,m^2$ $I = 1.4\,kW/m^2$

$= 1400\,W/m^2$

(1) $I = \frac{1}{2} c \varepsilon_0 E_0^2 \Rightarrow E_0 = \sqrt{\frac{2I}{c\varepsilon_0}}$

$$E_0 = \sqrt{\frac{2 \times 1400 \frac{W}{\cancel{m^2}}}{(3.0\times10^8)\,m/s\,(8.85\times10^{-12}\,C^2/N\,\cancel{m^2})}}$$

$$E_0 = \sqrt{\frac{2800\,W}{0.002655\,m/s\cdot C^2/N}} \Rightarrow E_0 = 1026.94\,\frac{N}{C}$$

$$E_0 = 1.02\times10^3\,\frac{N}{C}\ //$$

(2) All radiation is absorbed,

so $P = \frac{I}{c}$

$$F_R = PA = \frac{IA}{c} \Rightarrow F_R = \frac{1400\frac{W}{\cancel{m^2}}\cdot 2500\,\cancel{m^2}}{3.0\times10^8\,m/s}$$

$$F_R = 0.012\,\frac{W}{m/s} = 0.012\,N\ //$$

(3) $F_G = \frac{GMm}{r^2} \Rightarrow F_G = 6.67\times10^{-11}\frac{N}{\cancel{m^2}/\cancel{kg^2}}\cdot\frac{5.9\cdot10^{24}\,kg\cdot 419000\,\cancel{kg}}{(10^{11}\,\cancel{m})^2}$

$$F_G = 0.016\,N\ //$$

(4) $F_G = 0.016\,N$ and $F_R = 0.012\,N$,

so the gravitational force is stronger.